Sample tasks from:

Algebra Assessments Through the Common Core (Grades 6-12)

A resource from
The Charles A Dana Center at
The University of Texas at Austin 2011

About the Dana Center Assessments

More than a decade ago, the Charles A. Dana Center began work on collections of assessment tasks that could be used by teachers at many grade levels to enable them to assess student learning continually as they enacted mathematics instruction. These assessments, developed in collaboration with mathematics educators, are designed to make clear to teachers, students, and parents what is being taught and learned about the most central mathematical concepts at each grade or in each course. Published by the Dana Center as a series of books (available for order here: http://www.utdanacenter.org/products/math.php), these collections of assessments eventually encompassed tasks for middle school, Algebra I, Geometry, and Algebra II.

These tasks have been used and tested by tens of thousands of educators and their students. Now the Dana Center has published a selection of these tasks in new editions: 118 tasks in Algebra Assessments Through the Common Core (Grades 6-12) and 57 tasks in Geometry Assessments Through the Common Core (Grades 6-12).

The alignment to the Common Core State Standards for Mathematics of 164 existing Dana Center mathematics assessment tasks, and the development of 11 new tasks aligned to the standards, was made possible by a grant from Carnegie Corporation of New York and from the Bill and Melinda Gates Foundation. (Gates Foundation grant number OPP-48458 and Carnegie Corporation of New York grant number B 8312.) The statements made and views expressed are solely the responsibility of the authors.

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To support the adoption of the Common Core State Standards for Mathematics, the Dana Center is pleased to make available here two sample tasks from *Algebra Assessments Through the Common Core (Grades 6-12)*: “Mosaics” and “Paintings on a Wall.”

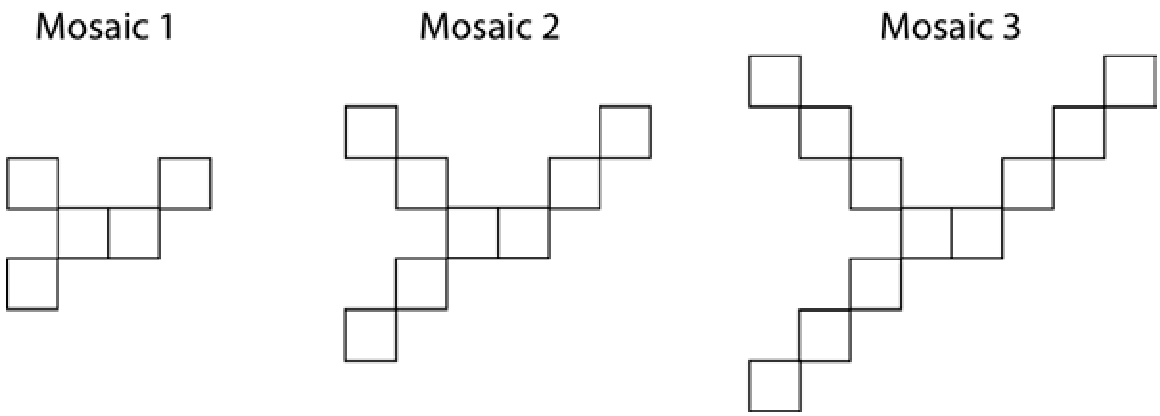
About Mosaics

Standard for Mathematical Practice #4 calls for students to model with mathematics. Students should be using the mathematics they know to solve problems that arise in everyday life. This task presents a situation in which tiles are used to create mosaics in a growing pattern. Students are asked to represent the relationship between the mosaic number and the number of tiles in the mosaic using multiple mathematical representations, making connections between the situation and the mathematical representations clear. Students use their models to answer questions and make predictions about the situation, and adjust their models based on changes in the situation.

About Paintings on a Wall

Standard for Mathematical Practice #5 calls for students to use appropriate tools strategically. Students should consider different tools available and decide when those tools might be helpful in solving a problem. In this task, students solve a problem using algebraic methods and technology, and then explain their solutions. Students can choose which representation to use in their explanations and therefore must consider which tools will be most useful in their explanations.

Mosaics



Reuben learned in art class that a mosaic is made by arranging small pieces of colored material (such as glass or tile) to create a design. Reuben created a mosaic using tiles, then decided on a growing pattern and created a second and third mosaic. Reuben continued his pattern by building additional mosaics. He counted the number of tiles in each mosaic and then represented the data in multiple ways. He thinks he sees a relationship between the mosaic number and the total number of tiles in the mosaic.

1. Represent Reuben's data from the mosaics problem in at least three ways, including a general function rule, to determine the number of tiles in any mosaic.

2. Write a description of how your rule is related to the mosaic picture. Include a description of what is constant and what is changing as tiles are added.

3. How many tiles would be in the tenth mosaic? Use two different representations to show how you determined your answer.

4. Would there be a mosaic in Reuben's set that uses exactly 57 tiles? Explain your reasoning using at least one representation.

5. In Reuben's mosaic, there are 2 tiles in the center. How would the function rule change if the center of the mosaic contained 4 tiles instead? Explain your reasoning using two different representations.

CCSS Content Task

(7.EE) **Solve real-life and mathematical problems using numerical and algebraic expressions and equations.**

4. Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.

  a. Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. *For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?*

(8.EE) **Analyze and solve linear equations and pairs of simultaneous linear equations.**

7. Solve linear equations in one variable.

  b. Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.

(8.F) **Use functions to model relationships between quantities.**

4. Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial

Scaffolding Questions

- Build or draw the next two mosaics. How many tiles are in each?
- Make a table to record your data, including a process column. What remains constant in the relationship between the mosaic number and the number of tiles in your table? What changes?
- Make a graph of the information from the table. How does your graph illustrate what is constant and what is changing in the problem situation?
- How would you describe the relationship between the mosaic number and the number of tiles in each mosaic?
- Which quantity depends on another quantity in this problem?
- What is the rate of change for tiles with respect to the mosaic number?
- On the graph for this problem situation, should a line connect the points? Explain why or why not.
- Develop a rule to determine the number of tiles in the *n*th mosaic. What is constant and what is changing in your rule?
- What is an appropriate domain and range for the mosaics situation?
- What are the connections among the picture, verbal description, table, and graph?

Sample Solutions

1. Represent Reuben's data from the mosaics problem in at least three ways, including a general function rule, to determine the number of tiles in any mosaic.

   Reuben's data can be represented with a verbal description, graph, table, and function rule.

   Verbal Description:

   One way to look at the mosaic is to see the 2 tiles in the center of the mosaic as the base. For each new mosaic, add 3 outside tiles.

Graph:

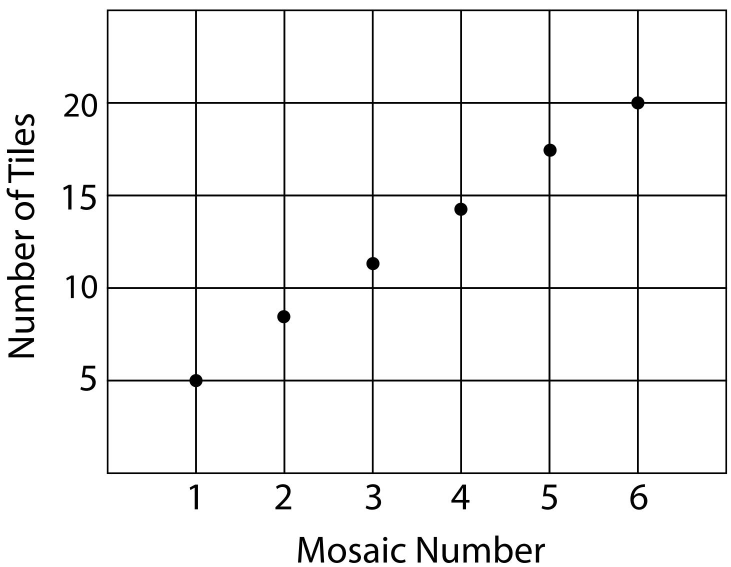


Table:

Mosaic Number	Process	Number of Tiles
1	2 + 3(1)	5
2	2 + 3(2)	8
3	2 + 3(3)	11
4	2 + 3(4)	14
5	2 + 3(5)	17
6	2 + 3(6)	20
…		
n	2 + 3(*n*)	

Function Rule:

From the table, we can find the rule for the number of tiles, T, that is 2 plus 3 times the mosaic number, n, or $T = 2 + 3n$.

value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.

(A-CED) **Create equations that describe numbers or relationships**

1. Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*

2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

(A-REI) **Understand solving equations as a process of reasoning and explain the reasoning**

1. Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

(A-REI) **Solve equations and inequalities in one variable**

3. Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

(F-BF) **Build a function that models a relationship between two quantities**

1. Write a function that describes a relationship between two quantities.*

   a. Determine an explicit expression, a recursive process, or steps for calculation from a context.

(F-LE) **Construct and compare linear, quadratic, and exponential models and solve problems**

2. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

(F-LE) **Interpret expressions for functions in terms of the situation they model**

5. Interpret the parameters in a linear or exponential function in terms of a context.

CCSS Additional Teacher Content

(6.EE) **Represent and analyze quantitative relationships between dependent and independent variables.**

9. Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and

independent variables using graphs and tables, and relate these to the equation. *For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation d = 65t to represent the relationship between distance and time.*

(F-IF) **Interpret functions that arise in applications in terms of the context**

5.Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function*.*

Standards for Mathematical Practice

2. Reason abstractly and quantitatively.

4. Model with mathematics.

2. Write a description of how your rule is related to the mosaic picture. Include a description of what is constant and what is changing as tiles are added.

   The function rule is based on the picture of 2 tiles in the center and 3 tiles added for each new mosaic number. As tiles are added, the 2 tiles in the middle are constant, and the number of tiles we add each time stays the same. What changes is the mosaic number. The total number of tiles depends on the mosaic number.

   From the graph you can see that for every new mosaic, the number of tiles increases by 3. If we followed the pattern in the graph back to Mosaic 0, we would have the 2 tiles in the middle. We can also see the constant rate of change: For each new mosaic number, the number of tiles increases by the constant 3.

   We can see the relationship between the mosaic picture and the table: The 2 tiles in the center remain constant, and the 3 tiles added for each new mosaic are constant. What changes is the mosaic number. The total number of tiles depends on the mosaic number. In every representation, we see that the total number of tiles is the 2 you start with plus 3 tiles multiplied by the mosaic number.

   Alternative Solution:

   Some students may see the 5 tiles in the first mosaic as the base. The first mosaic then consists of only the base. For subsequent mosaics, students add 3 tiles each time (or 3 times 1 less than the mosaic number). If this is how students see the pattern, then the process column in the table below will change to 5 + 3(mosaic number – 1).

   - Suppose Susan's process is shown in the table below. What is her original mosaic pattern and how does it change? How does this compare to Reuben's pattern?

Mosaic Number	Process	Number of Tiles
1	5 + 3(0)	5
2	5 + 3(1)	8
3	5 + 3(2)	11

Susan's function rule would be $T = 5 + 3(n - 1)$, where n is the mosaic number. She saw the original mosaic base consisting of 5 tiles and then began adding 3 tiles starting with the second mosaic. She gets the same number of tiles for each mosaic number as Reuben, but she saw the pattern in a different way.

> **Note:** This activity provides an opportunity to explore what is the same and what is different about the two rules. The two rules, though different, are equivalent; that is, $5 + 3(n - 1) = 2 + 3n$, but they represent two different ways of seeing the mosaic pattern. It is more important here for students to connect the way they "see" the pattern with the verbal description, rule, table, and graph. Although there is the opportunity here to illustrate the distributive property and combining like terms, these are not the objectives of the lesson. You may choose to revisit this activity later with a different focus.

3. How many tiles would be in the tenth mosaic? Use two different representations to show how you determined your answer.

   We can find how many tiles are in the tenth mosaic using the graph, table, or function rule.

Graph:

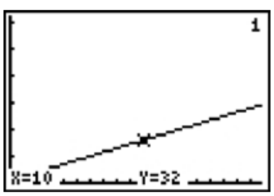
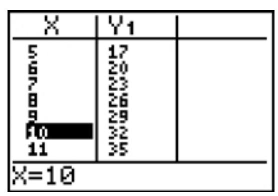


Table:



Function Rule:

$T = 2 + 3(10)$ or 32 tiles

4. Would there be a mosaic in Reuben's set that uses exactly 57 tiles? Explain your reasoning using at least one representation.

   To determine if there is a mosaic that has 57 tiles, we need to find the mosaic number that corresponds to 57 tiles. We can look at an extended table to determine the mosaic number, or we can look at the graph.

X	Y1
15	47
16	50
17	53
18	56
19	59
20	62
21	65

X=18

In either case, we see that since the mosaic number must be a whole number, there is not a table entry or a point on the graph corresponding to exactly 57 tiles. We can also set up an equation that arises from the function situation and solve it.

$57 = 2 + 3n$

$55 = 3n$

$n = \frac{55}{3} = 18\frac{1}{3}$

Note: You may have students who notice this possibility: If you split one tile into three equal parts and add them to each end, they could use 57 tiles.

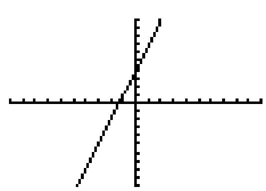
Since n must be a whole number, there would not be a mosaic with exactly 57 tiles.

5. In Reuben's mosaic, there are 2 tiles in the center. How would the function rule change if the center of the mosaic contained 4 tiles instead? Explain your reasoning using two different representations.

   If the center of the mosaic contained 4 tiles, the data in the table would change. Each y-value would be 2 greater than the y-value representing the number of tiles in the original mosaic. Each point in the graph would be translated up by 2 units.

X	Y1
1	7
2	10
3	13
4	16
5	19
6	22
7	25

X=1



Generating a new rule, we see the constant would be 4. The new rule would be $T = 4 + 3n$.

Extension Questions

- If the function rule was $T = 2 + 4n$, describe the first two mosaics and the general rule.

  There would be 2 tiles in the center and 1 tile on each of the four corners for the first mosaic. The second mosaic would have 2 tiles in the center and 2 tiles at each of four corners. The general rule means that there are 2 tiles in the center and 4 tiles added for each new mosaic.

- Draw the first three mosaics for the following function rules. Make a table for each and then graph the functions on the same axes. Describe similarities and differences.

  A. $x + 3 = y$

  Picture:

  Draw Mosaic 1 with 3 tiles in the center and 1 tile on one corner, Mosaic 2 with 3 tiles in the center and 2 tiles on one corner, and Mosaic 3 with 3 tiles in the center and 3 tiles on one corner.

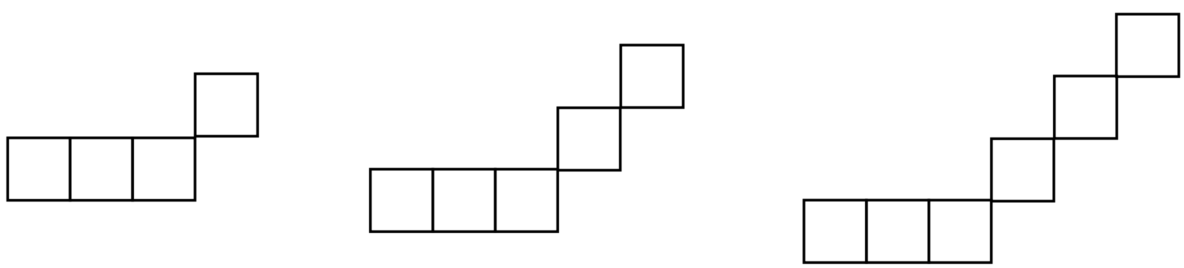


Table:

Mosaic Number	Number of Tiles
1	4
2	5
3	6

B. $4x + 3 = y$

Picture:

Draw Mosaic 1 with 3 tiles in the center and 1 tile on each of the four corners, Mosaic 2 with 3 tiles in the center and 2 tiles on each of the four corners, and Mosaic 3 with 3 tiles in the center and 3 tiles on each of the four corners.

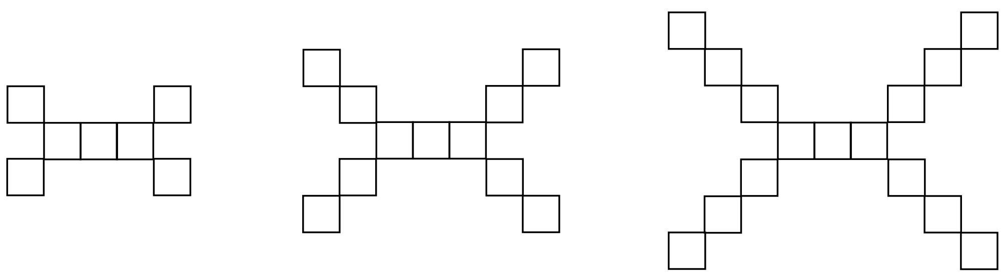


Table:

Mosaic Number	Number of Tiles
1	7
2	11
3	15

C. $4x + 6 = y$

Picture:

Draw Mosaic 1 with 6 tiles in the center and 1 tile on each of the four corners, Mosaic 2 with 6 tiles in the center and 2 tiles on each of the four corners, and Mosaic 3 with 6 tiles in the center and 3 tiles on each of the four corners.

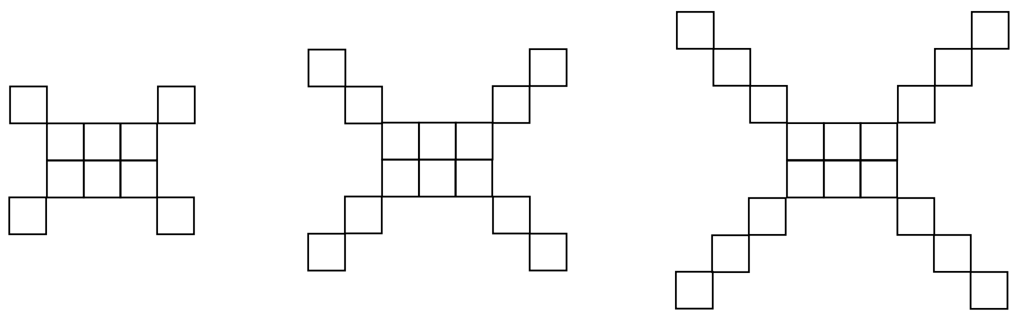
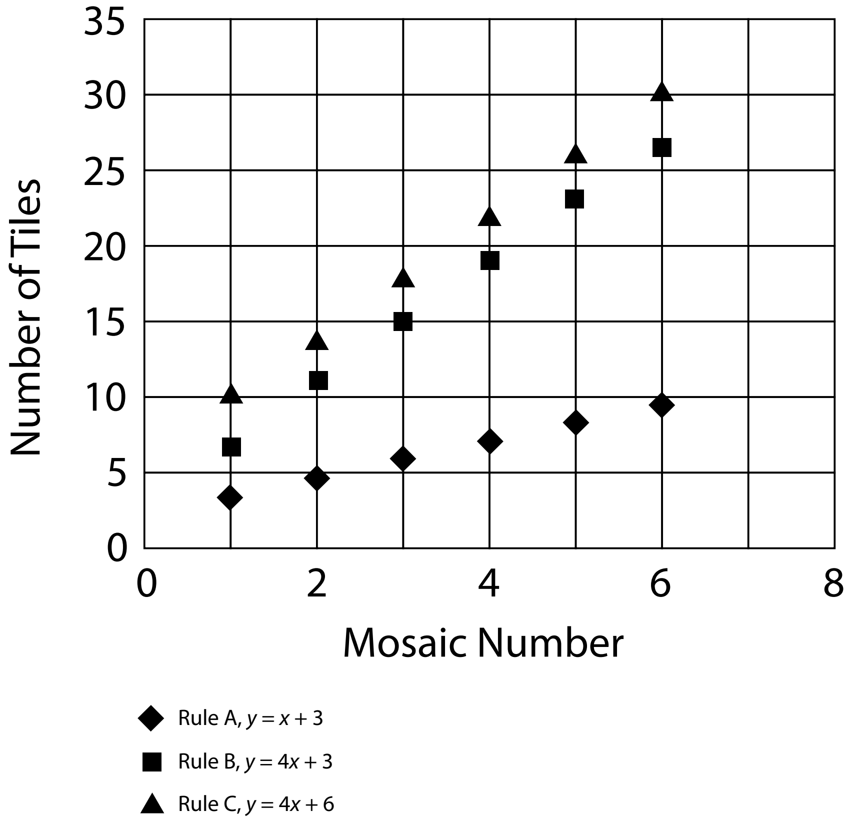


Table:

Mosaic Number	Number of Tiles
1	10
2	14
3	18

Functions graphed together

When you graph them together on the same set of axes, you can see that all of the rules are linear realtionships and that the graphs of rules B and C are parallel because they have the same slope.


35
30
25
20
15
10
5
0
0
2
4
6
8
Number of Tiles
Mosaic Number
Rule A, $y = x + 3$
Rule B, $y = 4x + 3$
Rule C, $y = 4x + 6$

Paintings on a Wall

In order to optimize the viewing space for its patrons, a museum has placed size restrictions on rectangular paintings that will be hung on a particular wall.

The perimeter of a painting must be between 64 inches and 100 inches, inclusive. The area of the painting must be between 200 square inches and 500 square inches.

1. You are in charge of determining possible perimeter and area combinations for paintings to be hung on the wall. Write inequalities to describe the perimeter and area restrictions in terms of the length and the width of the rectangles. Graph the resulting system.

2. Algebraically and with technology, determine the vertices of the region defined by the inequalities in problem 1.

3. Describe the location of the points on your graph where the dimensions of the painting result in each of the following:

   a. The perimeter and area are acceptable.
   b. The perimeter is too short or too long, but the area is acceptable.
   c. The perimeter is acceptable, but the area is too small or too large.
   d. Neither the perimeter nor the area is acceptable.

   Explain how you arrived at your responses.

Notes

CCSS Content Task

(A-CED) **Create equations that describe numbers or relationships**

3. Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.*

(A-REI) **Solve equations and inequalities in one variable**

4. Solve quadratic equations in one variable.

b. Solve quadratic equations by inspection (e.g., for x2 = 49), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as a ± bi for real numbers a and b.

(A-REI) **Represent and solve equations and inequalities graphically**

11. Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); find the solutions approximately,

Scaffolding Questions:

- What information is known?
- What are you expected to show?
- What compound inequality can you write to describe the restrictions on the perimeter?
- What compound inequality can you write to describe the restrictions on the area?
- How can you use the inequalities to write functions that describe the boundaries of the region containing acceptable values for width and length of a painting?
- Describe the process you might use to graph the compound inequality.
- What functions must be graphed to model a combined inequality such as $2 \le x + y \le 5$?
- What kind of systems must you solve to get the vertices of the boundary?
- What algebraic method will you use to solve each system? Why?
- Point to each region in the plane determined by the boundary functions and describe whether the perimeter and area meet the requirements in that region.

Sample Solutions:

1. Let l = the length in inches of a painting and w = the width in inches of the painting.

   Since the perimeter of the painting must be between 64 inches and 100 inches, we know that

   $$64 \le 2(l + w) \le 100$$

   which gives

   $$32 \le l + w \le 50$$
   $$32 - w \le l \le 50 - w$$

Since the area of the painting must be between 200 square inches and 500 square inches, we have

$$200 \le lw \le 500$$

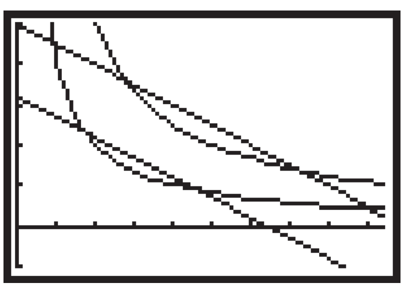
which gives

$$\frac{200}{w} \le l \le \frac{500}{w}$$

On the graphing calculator, the length will be represented by Y and the width will be represented by x.

Function Rule:	Calculator Rule:
$l = 32 - w$	$Y_1 = 32 - x$
$l = 50 - w$	$Y_2 = 50 - x$
$l = \frac{200}{w}$	$Y_3 = \frac{200}{x}$
$l = \frac{500}{w}$	$Y_4 = \frac{500}{x}$

Here is the calculator graph of the system with window settings $0 \le x \le 47$, $-10 \le y \le 50$:



The region that is the solution set to the system of inequalities is the closed region between the two lines and the two curves.

2. We use the intersect feature of the calculator to determine the coordinates of the vertices. Starting with the upper left vertex and moving counterclockwise around the region, they are:

The intersection of Y_2 and Y_3: A(4.38, 45.62)
The intersection of Y_1 and Y_3: B(8.52, 23.48)

e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where f(x) and/or g(x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.*

12. Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

(F-IF) **Interpret functions that arise in applications in terms of the context**

4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.**

Notes

Interpret functions that arise in applications in terms of the context

(F-IF.5) Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.**

Analyze functions using different representations

(F-IF.7) Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.*

d. (+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.

Build a function that models a relationship between two quantities

(F-BF.1) Write a function that describes a relationship between two quantities.*

b. Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model*.

Standards for Mathematical Practice
5. Use appropriate tools strategically.
6. Attend to precision.

The intersection of Y_1 and Y_3: C(23.48, 8.52)
The intersection of Y_2 and Y_3: D(45.62, 4.38)
The intersection of Y_2 and Y_4: E(36.18, 13.82)
The intersection of Y_2 and Y_4: F(13.82, 36.18)

To find the vertices algebraically, we solve the following systems by substitution:

$$Y_1 = 32 - x$$
$$Y_3 = \frac{200}{x}$$
$$\text{Let } Y_1 = Y_3$$
$$32 - x = \frac{200}{x}$$
$$32x - x^2 = 200$$
$$x^2 - 32x + 200 = 0$$
$$x = \frac{32 \pm \sqrt{(-32)^2 - 4 \cdot 1 \cdot 200}}{2}$$
$$x = 23.48 \text{ or } x = 8.52$$

This gives the *x*-coordinate for vertices B and C.

Substitute for *x* in Y_1 to get the *y*-coordinates.

$$Y_2 = 50 - x$$
$$Y_3 = \frac{200}{x}$$
$$\text{Let } Y_2 = Y_3$$
$$50 - x = \frac{200}{x}$$
$$50x - x^2 = 200$$
$$x^2 - 50x + 200 = 0$$
$$x = \frac{50 \pm \sqrt{(-50)^2 - 4 \cdot 1 \cdot 200}}{2}$$
$$x = 45.62 \text{ or } x = 4.38$$

This gives the *x*-coordinate for vertices D and A.

Substitute for *x* in Y_2 to get the *y*-coordinates.

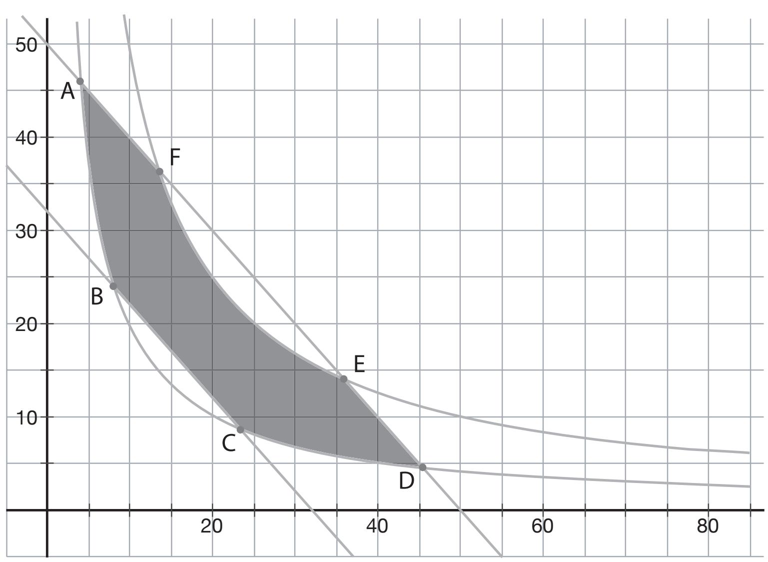
$$Y_2 = 50 - x$$
$$Y_4 = \frac{500}{x}$$
$$\text{Let } Y_2 = Y_4$$
$$50 - x = \frac{500}{x}$$
$$50x - x^2 = 500$$
$$x^2 - 50x + 500 = 0$$
$$x = \frac{50 \pm \sqrt{(-50)^2 - 4 \cdot 1 \cdot 500}}{2}$$
$$x = 36.18 \text{ or } x = 13.82$$

This gives the *x*-coordinate for vertices E and F.

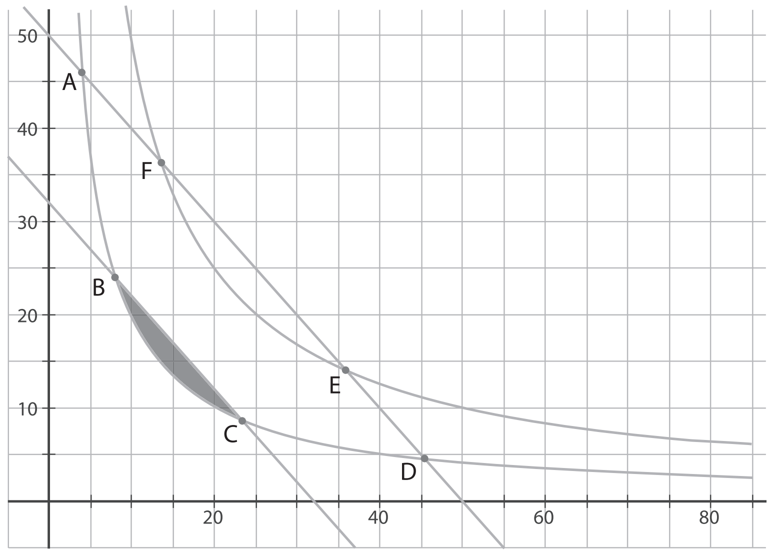
Substitute for *x* in Y_2 to get the *y*-coordinates.

3. Remember that the coordinates *(x, y)* of the points in the plane represent (width, length) of the painting. Therefore, we can consider points only in the first quadrant.

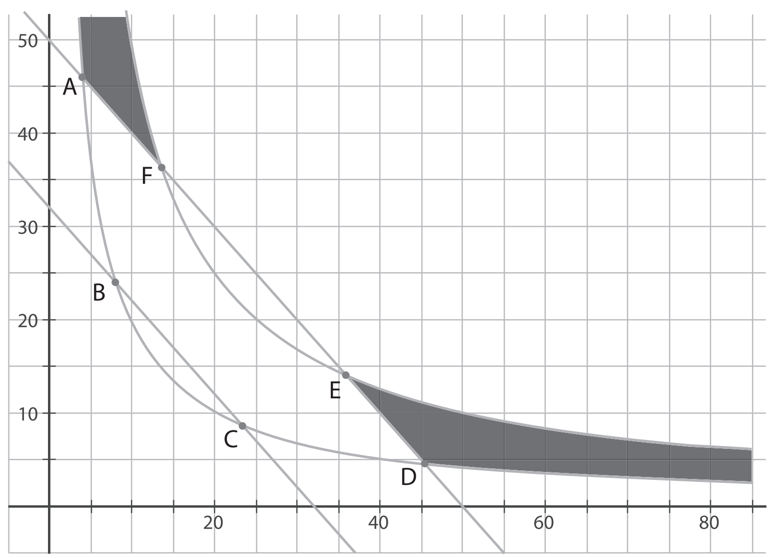
   a. The region where the perimeter and area are acceptable is the closed region between the lines and the curves.



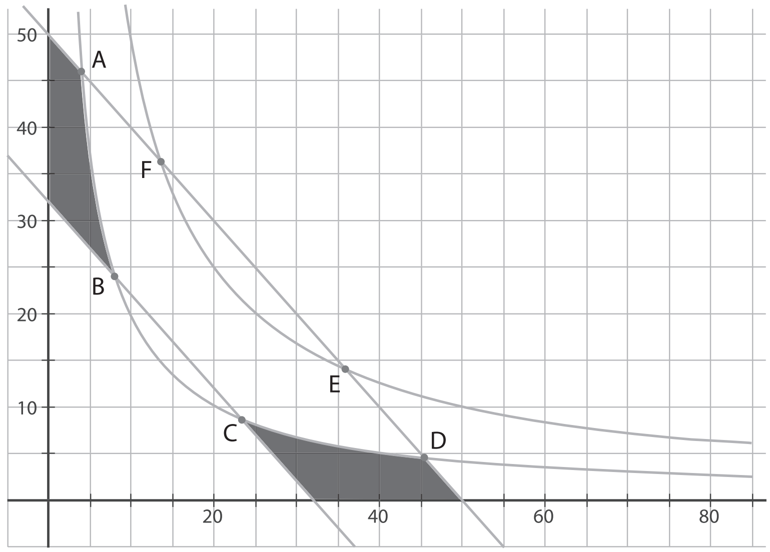
   b. For the perimeter to be too short and the area acceptable, we need first quadrant points below the line $y = 32 - x$ but between or on the curves.



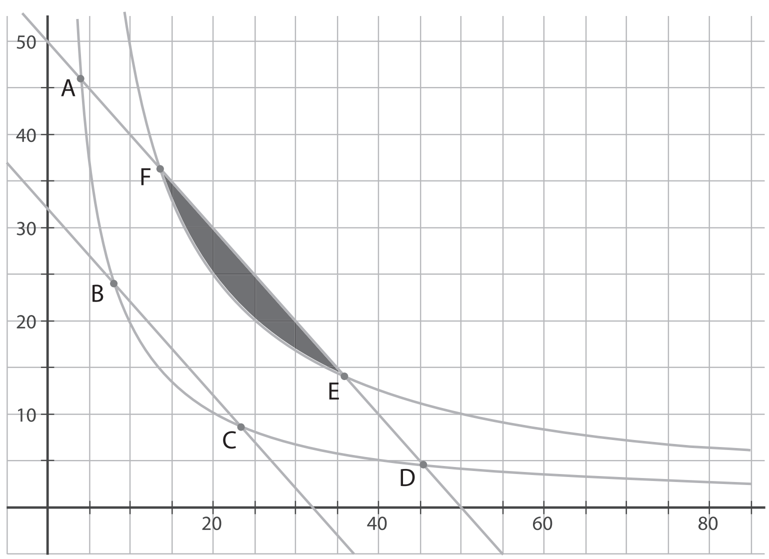
For the the perimeter to be too long and the area acceptable, we need first quadrant points above the line $y = 50 - x$ but between or on the curves.



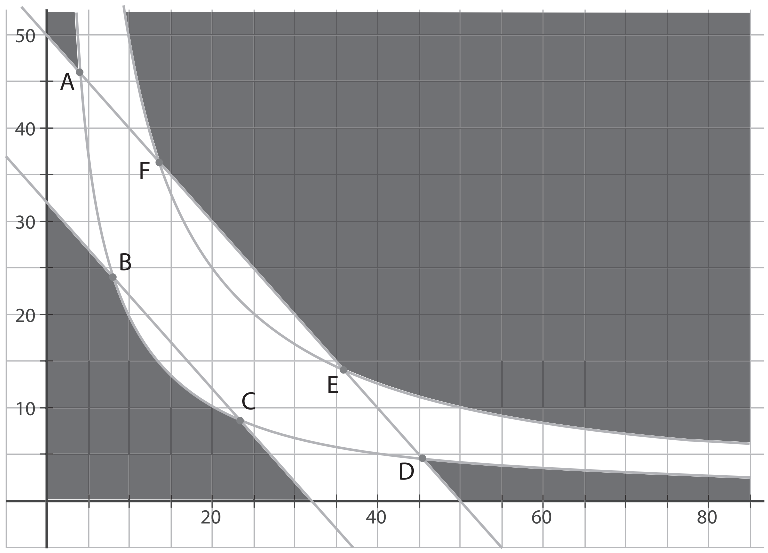
c. For the perimeter to be acceptable and the area too small, we need first quadrant points between or on the lines but below the curve $y = \frac{200}{x}$.



For the perimeter to be acceptable and the area too large, we need first quadrant points between or on the lines but above the curve $y = \dfrac{500}{x}$.



d. If the perimeter and area are both unacceptable, we need first quadrant points both outside the lines and the curves.



Extension Questions:

- How is the region representing acceptable dimensions for a painting different from regions you encountered in linear programming problems?

  The boundaries are not all defined by linear functions. Three boundaries are segments, defined by the two linear (perimeter) functions. Three boundaries are defined by the two reciprocal (area) functions.

- Which vertex or vertices minimize perimeter and area, subject to the restrictions? How do you know?

  To minimize both perimeter and area you must be at the intersection of $Y_1 = 32 - x$ and $Y_3 = \frac{200}{x}$ *because these give the lower limits on perimeter and area. This is vertex B or C. The painting can have dimensions 8.52 inches by 23.48 inches. To maximize both perimeter and area, you must be at the intersection of* $Y_2 = 50 - x$ and $Y_4 = \frac{500}{x}$ *because these give the upper limits on perimeter and area. This is vertex E or F. The painting can have dimensions 36.18 inches by 13.82 inches.*

- Does the graph have any symmetry?

  Yes. It is symmetric with respect to the line y = x.

- Suppose the area restriction were replaced with the restriction that the diagonal of a painting must be between 25 inches and 40 inches in length. How will this change your responses to the previous two questions?

  The restrictions on the perimeter stay the same, but we must replace area restrictions with diagonal restrictions.

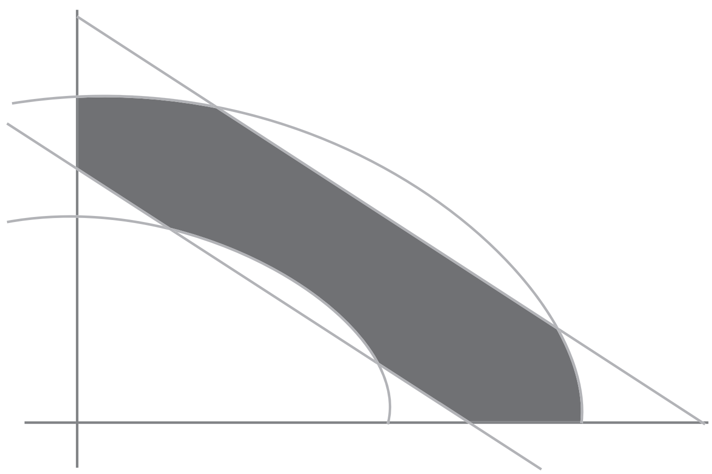
Let d = the length of the painting's diagonal in inches. Then, since $d^2 = l^2 + w^2$,

$$25 \le d \le 40 \Rightarrow 25^2 \le d^2 \le 40^2$$
$$25^2 \le l^2 + w^2 \le 40^2$$
$$25^2 - w^2 \le l^2 \le 40^2 - w^2$$
$$\sqrt{25^2 - w^2} \le l \le \sqrt{40^2 - w^2}$$

The area boundary equations are replaced with

$$l = \sqrt{25^2 - w^2} \text{ and } l = \sqrt{40^2 - w^2}$$

The graph of the region representing acceptable dimensions is the first quadrant region between the lines and between the semi-circles. It does not include points on the axes, since that would give you zero width or zero length.



To find where the perimeter lines and diagonal circles intersect, we solve the systems consisting of Y_1 *and* Y_3 *and of* Y_2 *and* Y_4*. For example,*

$$Y_1 = 32 - x$$
$$Y_3 = \sqrt{25^2 - x^2}$$
$$\text{Let } Y_1 = Y_3$$
$$32 - x = \sqrt{25^2 - x^2}$$
$$x^2 - 64x + 32^2 = 25^2 - x^2$$
$$2x^2 - 64x + 32^2 - 25^2 = 0$$

Apply the quadratic formula to get x = 8.48 inches or x = 23.52 inches.

Similarly, we solve for the intersection of Y_2 and Y_4 *to get x = 11.77 inches or x = 38.23 inches.*

The vertex points are (8.48 , 23.52), (23.52, 8.48), (11.77, 38.23) and (38.23, 11.77).